Research on Learning Trajectories in Mathematics and Science

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Questions about Learning Trajectories

• What are they (and what are they not)?
• Is there a difference between LTs in math and science?
• How standardized are they for any particular topic?
• Why develop particular ones?
• How do you recognize indicators of robust LTs?
• What structures, theories, and methodologies apply?
• What use are they?
• What advice do we have for future directions?
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What is a learning trajectory?

- Focus is on learning as an active process of building and modifying ideas,
- Possible path or set of paths from prior knowledge to more sophisticated reasoning,
- Establish target,
- Establish base,
- Describe intermediate steps;
  - communal and individual successive reorganizations of content within an engineered instructional environment.
- States of “likelihood”, not regimented steps or psychological stages;
- Include predictable landmarks and obstacles
- Populates an unsettled relationship (messy middle) to authorized knowledge in a discipline, brought to local closure through instruction.
Learning Trajectory within a Conceptual Corridor

Confrey (2006) Design Studies Chapter
Cambridge Handbook of the Learning Sciences
What are they not?

- Logical deconstructions of prerequisites of a domain concept,
- Thought experiments about sequencing, absent empirical investigation of learning,
- A linear sequence of curricular tasks based solely on a literature review,
- A whole discipline
- Development au naturel.
How standardized are they for any particular topic?

They are:

• Likelihoods and common behaviors.
• Investigated and engineered (science and engineering based)
• Collectively built by students, scaffolded by teachers, and emergent
• Task- and tool-dependent
• Dependent on significant opportunities for interaction, discussion, and divergent thinking
• Grounded in literature explaining similarities and differences
Are there differences between the nature of LTs in math and science?

• Science LP Definitions:

• “...empirically grounded and testable hypotheses about how students’ understanding of, and ability to use, core scientific concepts and explanations and related scientific practices grow and become more sophisticated over time, with appropriate instruction” (p. 8)
Are there differences between the LTs in math and science?

- Progressions tend to be in science and trajectories in math [nuances in definition of “progressions” esp. in science]
- Progressions tend to be bigger chunks of material
- Some distinguish progressions (curriculum) from trajectories (developmental sequences) [Clements and Sarama; Lehrer]
- Structure of LPs reflects the structure of approved and “resolved” knowledge in the scientific field.
- Many science LPs have huge meta-theme as “the big idea”
- Variation in emphasis on student- emergence about models
  - Some ascribe how students come to know predefined models
  - Others emphasize how children bring informal knowledge and experience and link to formal models progressively
Distinction in science LPs

- Distinction Curriculum and instruction type vs. cognition and instruction type (Shavelson, 2009)

- Curriculum and instruction type: Sequences of concepts based on logical analyses of big ideas in science to serve as foundations for curricular units.

- Cognition and instruction type: Maps out a progression for how students come to understand scientific concepts from the naïve everyday ideas to scientific accurate.

“This [curriculum and instruction] type of learning progression is not empirically validated and is highly dependent on context, meaning that the curriculum is being used will influence how students will perform with respect to the learning progression. Songer and Gotwals (2012) among others have argued that learning progressions [cognition and instruction type] should not represent content alone, but should be in some way integrated with scientific practices that support student learning of science ideas” Furtak 2012
An example from math

- **Equipartitioning** (or *splitting*) indicates cognitive behaviors that have the goal of producing equal-size groups (from collections) or pieces (from continuous wholes) as “fair shares” for each of a set of individuals.

- Equipartitioning is *not* breaking, fracturing, fragmenting, or segmenting in which there is the creation of unequal parts.

- Target understanding: Generalization that *a* items shared among *b* people is the same as \( a/b = a \div b \) (CC Standard 5.NF.3)
Why develop an equipartitioning learning trajectory?

• multiplication and division unduly delayed until 3rd grade as derived from addition and subtraction,
• remarkable competence evident in fair sharing and the discourse surrounding it,
• different roots postulated for counting and splitting from a 3-year teaching experiment (grade 3-5) (Confrey, 1991),
• different cases of fair sharing researched separately lacking integration, and
• weak performance on the target (a divided by b = a/b) as a big idea
# The Equipartitioning Learning Trajectory and the Standards

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Close listening and observing
Equipartitioning Wholes
Equipartitioning Wholes
Example of New Construct: PEEQ

• Property of Equality of Equipartitioning (PEEQ): If two congruent shapes are each split for the same number of persons, then the size of the shares from one of the shapes is equal to the size of the shares from the other shape, regardless of the shape of the shares.
Read the following problem, and predict how

Ann and Beth have brownies that are the same size. Both children cut their brownies as shown in the drawings below. A piece of Ann’s brownie is shaded in on the left and a piece of Beth’s brownie is shaded on the right.

Ann’s Brownie

Beth’s Brownie

Is the piece of Ann’s brownie bigger, smaller, or the same size as the piece of Beth’s brownie? (Circle One)

Bigger

Smaller

The Same Size As
Student demonstrates equivalent results of two strategies

*transitivity argument*: shares from both splits are still \( \frac{1}{2} \) of the same size brownie)...

...or shows equivalence by *compensation* or *decomposition*
How do you recognize indicators of robust LTs?

- **Synthesis of Literature**, integrating disparate cases
  - Sharing Collections (Hunting & Sharpley, 1991; Pepper, 1991);
  - Sharing Wholes (Piaget, 1971; Pothier & Sawada, 1983; Confrey et al., 2009; Empson & Turner, 2006);
  - Sharing Multiple Wholes (Charles & Nason 2000; Lamon 1966)

- **Identifying a foundation** for whole trajectory: three criteria for equipartitioning-
  - equal-sized,
  - correct number,
  - exhaust the whole (Piaget)

- **Finding new constructs** –
  - re-assembly as the reverse of partitive vs. quotative/measurement division. “Times as many” and “1/nth of”
How do you recognize indicators of robust LTs?

- Utilizing collective socio-mathematical activity: -- naming and justifying (“times as many or much”)

- Leveraging misconceptions: Composition of splits – seeing how repeated splitting played out

- Encountering Surprises: Easier to do 7 shared among 4 than 4 shared among 7
Look-fors

• A robust learning trajectory should include: new constructs, surprises, options, testable hypotheses, and data
What kinds of structures, evidence, and validation apply?

• Theory
• Methodology
• Evidence Claims
• Utility arguments
What theories?

• Generally constructivist or soci-cultural or both
• Emergent instructional theories that scaffold content (not just supporting learning but fostering it)
• Grand and humble theories that do work
What are the phases of LT Research and Development?

- Selecting the target (big idea, difficult to learn, affected by new tools or ways of thinking)
- Literature review and/or synthesis
- Conducting initial clinical interviews
  - Design tasks, sequences, and tools
- Short design cycles with instructional supports
  - Pre- and post assessments
  - Materials development, tools, and/or assessment to elicit thinking
    - Establishing classroom norms
  - Creating data collection methods
- Longer design studies

- Scale up
Design Study Methodology

Five features of Design Studies:

1. generating theory
2. implementing highly interventionist strategies, 
3. iterative design across three phases of the study (planning, implementation, and retrospective analysis)
4. placing “humble” domain-specific theories in harm’s way (being willing to revise or reject depending on observations and results)
5. requiring theories to be accountable to the activity of design→requiring them to successfully do work if they are to survive.

Cobb, Confrey, diSessa, Lehrer & Schauble 2003)
Item Validation for assessing LTs

- Design for item validation
  - ~4800 children across K-7
  - 33 test forms, with overlapping items for calibration
Method: Item Analysis

Wright Map from IRT analysis: Grades 2-3-4-5-6

Each X represents 10 student cases.

KEY
Strategies and representations: Red
Mathematical Practices: Red
Emergent Properties: Green
Generalizations: Blue

Numbers indicate the item number;
Colors represent location of item in the framework for understanding.
IRT: difficulty estimates—proficiency levels
To what use has the EQP LT been put?

- LTBI-Learning Trajectory-based Instruction project (NSF)
- MOOCs on equipartitioning (Maloney)
- CCSS—incorporated principles of EQP
- TurnOnCCMath: LTs for K-8 mathematics to interpret CCSS
- Licensing to commercial firm for curriculum product development
- Gates Foundation Learning Map work
What use are LTs?

Valued for:
• Informing standards development
• Designing professional development (Clements & Sarama, Wilson, Mojica, Learning Trajectory-Based Instruction)
• Developing formative assessment practices (Heritage et al., Furtak)
• Building diagnostic assessments (Confrey, Izsak, Lehrer/Wilson)
• Curriculum development (Clements et al., )
• Large-scale assessment (Alonzo et al)
Caveat on Professional Development Use

• “.. several of the teachers seemed to use the learning progressions simply as catalogs of misconceptions to be “squashed”, rather than drawing upon the developmental affordances offered by a learning progression.” (Futak, 2012)
Looking forward to a new genre

- Deeply digital learning systems
- Learning Maps and related assessment systems (supported by Gates)
- Examples: SUDDS, Dynamic Learning Maps, Enlearn, CRESST, GlassLabs, New Classrooms
- Facilitating navigation, depth of knowledge, and analytical activity

- Demo map
NSF’s role?

• LTs/LPs provide a foundation for STEM ed, right grain-size

• Need additional research in specific topic areas:
  – Less-investigated topics and big ideas,
  – high school topics: LTs require integration of multiple strands (such as RLCs)

• STEM Centers to support programmatic efforts including libraries of exemplars, means to leverage for applied uses, related tasks and assessments access
  – Library of how LT/LP work can be conducted programatically
  – Topic mini-centers (Smith, J., MSU: length and area)
  – Modelingdata.org: Rich Lehrer’s IES site is a great example (PW modeling)
NSF’s role?

• Should fit into a larger context of continuous school improvement and equity (Bryk and Gomez) and deeply digital strategies (Cyberlearning) with clear links to:
  – curriculum
  – Through-course or diagnostic assessment measures
  – Professional development
  – Targeted accelerated learning for low performers
References

References

References

• Questions?

• Thank you.